

2.6 Result (17) on page 38 tells us the multiplicity function for the two spin systems away from the equilibrium by amount δ , i.e. for the state when the first system has a spin excess

$$S_1 = \hat{S}_1 + \delta \quad (\hat{S}_1 \text{ is the equilibrium value})$$

(*) and the second one has

$$S_2 = \hat{S}_2 - \delta$$

Note that we have to have $S_1 + S_2 = \hat{S}_1 + \hat{S}_2 = S_{\text{tot}}$, because $S_1 + S_2$ is fixed at S_{tot} . That's why we have $+\delta$ and $-\delta$ in the equations above, rather than, say, $+\delta_1$ and $-\delta_2$.

Now, we know that the probability of any macrostate is proportional to its multiplicity, so from (17), the probability of the state described by (*) is

$$P(N_1, N_2, \delta) = \text{const} \exp\left(-\frac{2\delta^2}{N_1} - \frac{2\delta^2}{N_2}\right)$$

$N_1 = N_2 \equiv N/2$

$$\stackrel{|||}{P(\delta)} = \text{const} \exp\left(-\frac{4\delta^2}{N}\right) = \text{const} e^{-a^2\delta^2}, \quad a = \sqrt{\frac{4}{N}}$$

We want to find the probability that $|\delta|/N_1 > 10^{-10}$, i.e. that $-N_1 10^{-10} > \delta$ or $N_1 10^{-10} \leq \delta$, i.e. that $\delta < -\delta_0$ or $\delta > \delta_0$, where $\delta_0 = N_1 10^{-10} = 10^{22} 10^{-10} = 10^{12}$.

By symmetry this probability is twice the probability to have $\delta > \delta_0$, so

$$(1) \quad p = 2 \sum_{\delta=\delta_0}^{+\infty} P(\delta) \stackrel{\uparrow \text{standard approximation by an integral}}{=} 2 \int_{\delta_0}^{\infty} P(\delta) d\delta = 2 \text{const} \int_{\delta_0}^{\infty} e^{-a^2\delta^2} d\delta$$

Now, the total probability is always 1, so

$$(2) \quad 1 = \sum_{\delta=-\infty}^{+\infty} P(\delta) = \int_{-\infty}^{+\infty} P(\delta) d\delta = \text{const} \int_{-\infty}^{+\infty} e^{-a^2\delta^2} d\delta$$